
بسم الله الرحمن الرحيم

Some Notes Regarding Gharar Transactions: A Game Theory Approach

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Introduction

Prohibition or denial of Gharar is one of the major Islamic economic rules. In this regard, it is very important to study this rule and the consequences of Gharar transactions.

Furthermore, by using the modern economic tools, the logic and wisdom of this Islamic rule can be understood. The main purpose of this paper is to use the game theoretic approach to study the rule of prohibition of Gharar, and why these kinds of transactions are forbidden in Islam.

Definition of Gharar

Islamic scholars have defined Gharar as risk or uncertainty implying delusion and deception, exposed to be perished, what the end is unknown, thing which have deceptive appearance but inside is unknown.

In financial transactions, Gharar is the uncertainty, which is due to shortages of information or some deficiencies in contracts in a way that limiting and controlling the Gharar transactions is one way of risk management in Islam.

Definition of Gharar

The main source of the rule of prohibition of Gharar is the hadith from the prophet (ﷺ) ”نهى النبى عن بيع الغرر ” .

Foghaha have referred to this rule in their fatwas (فتاوى). However, there are differences in its aspects.

Definition of Gharar

Some are referring it to the conditions of transactions; some for instances of transactions; for some, Gharar transaction is the one that its existence is not known for sure or its quantity is unknown or the one, which there is no power for giving or receiving the deal. To summarize, one could say that Gharar is where there is the probability of loss in a transaction because of ambiguity in some aspects of the transaction, which itself is due ignorance and uncertainty in the transaction.

The Model

In this part, we are trying to model a hypothetical Gharar transaction and study the consequences of such a transaction for the society. As was mentioned, one of the aspects of Gharar is the ignorance about the deal from the quantitative and/or qualitative points of view. That is, a transaction in which its characteristics and quality at least for one side is not known and because of this ignorance, there is the probability of loss and this transaction could be considered as a Gharar transaction.

In this line of study, we have assumed a transaction in which the quality of the good is not known for one side, i.e. buyer.

The Model

Assumptions:

- there are two kinds of goods in this market, high quality good (H) and low quality good (L).
- (p) percent of the sellers are selling high quality goods and (1-p) percent are selling low quality goods.
- The sellers are well aware of the quality of their goods, while the buyers only know the probability distribution of it. That is we are facing an asymmetric information problem.
- When there is asymmetric information then comes the problem of transferring the information.
- The sellers are sending high price and low price to signal about the quality of their commodities. Thus the different strategies of the sellers are: asking for high price (HP) or asking the low price (LP). On the other hand the buyers have two options, accepting (A) the price or rejecting (R) it.

The Model

We assume that each seller obtains the same profit, , $\pi > 0$, from selling high quality good with high price and low quality good with low price.

The seller of high quality good always asks for high price – there is no reason for selling high quality good for low price. It is assumed that the seller can earn extra profit by selling low quality good with a high price $\pi'(\pi' > \pi)$.

We assume that each buyer gets the same utility , $u > 0$, when buying high quality good with high price and low quality good with low price.

The Model

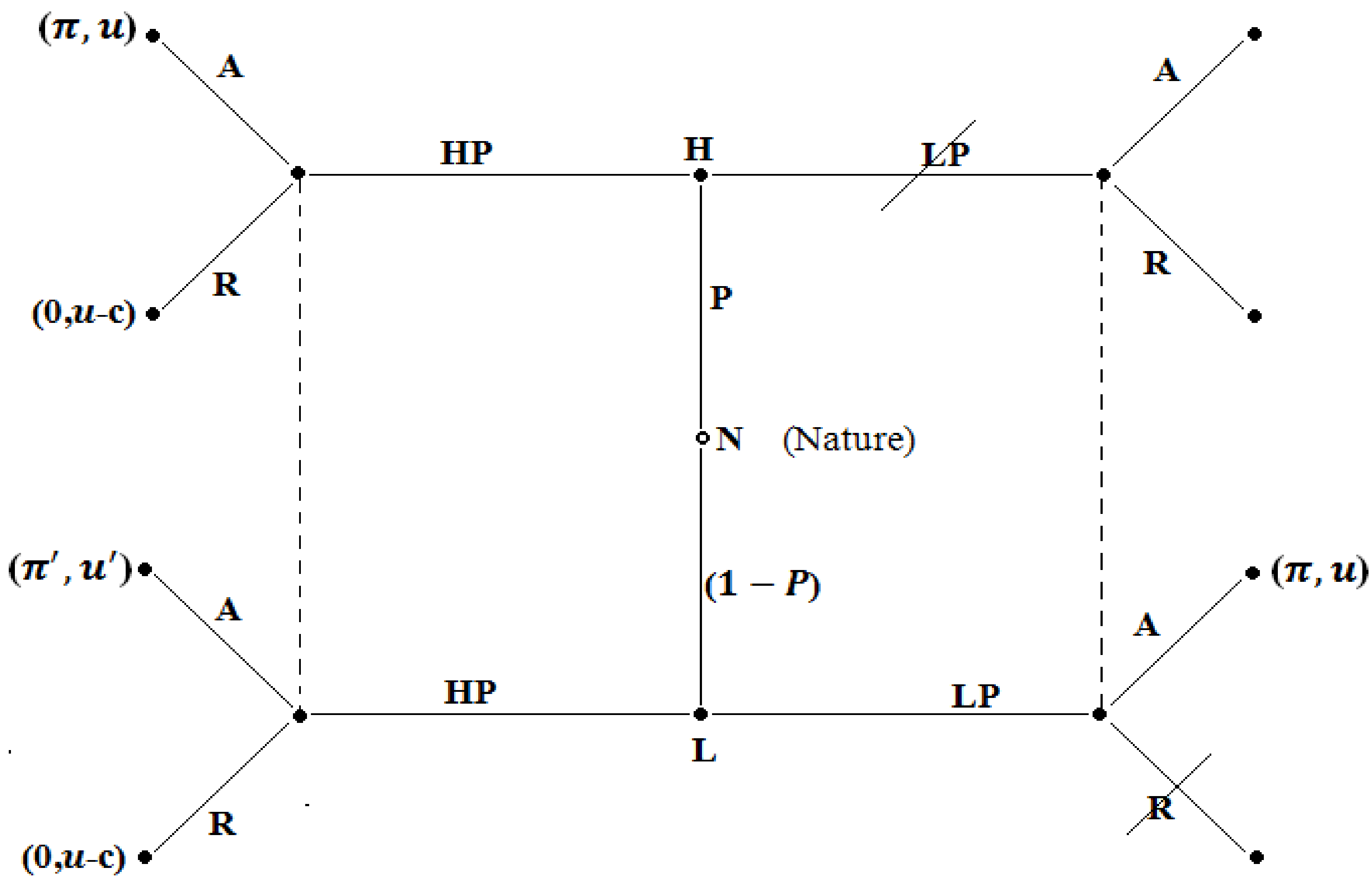
The buyer always accepts low price, there is no doubt regarding the quality of good in this situation. But for the high price, the buyer can accept or reject it. If the buyer does not accept the high price asked by seller, he/she should search for extra information to know about the quality of the good. Searching is costly, time and money which should be spend to obtain more information. This cost will give him/her the disutility by the amount of c. That is, in this situation the buyer's utility would be $(u-c)$.

Furthermore, the buyer will have the utility of u' $(u' < u)$ when accepting the high price for the low quality good.

The Model

It should be noted that $u'' = (u - c) > u'$, because otherwise searching for extra information would have been useless.

Under these assumptions we can model the situation as a strategic game in which the seller has two actions– asking high price or low price, and the buyer has two actions – accepting it or rejecting and spending more time to acquire extra information. Since there is conflict of interests between sellers and buyers and the flow of information is from the sellers to the buyers, we can show the game as an extensive form of signaling game, which is shown in diagram (1).



(π, u)
A
HP
H
LP
A
R
$(0,u\text{-c})$
R
P
N (Nature)
(π', u')
A
$(1 - P)$
(π, u)
A
HP
LP
L
R
$(0,u\text{-c})$
R

Diagram (1)

The Model

As can be seen from the diagram (1) according to the beliefs of the buyers, the nature determines the qualities of good to be high (H) or low (L) with the probabilities of (p) and (1-p) respectively. The seller knows his situation with certainty and signals the buyer, on the other hand, the buyer cannot extract with certainty the quality of good by receiving the signals from the seller, which is shown with dotted line in diagram (1). The payoffs are also shown; the first payoff is for the seller and the second is for the buyer.

The Model

Now the point is that if in this society the sellers were honest and not looking for Gharar transactions and their signals were right and correct – high price for high quality good and low price for low quality good– and the buyers believing their honesty, in this situation the profit of the seller is π and the utility of the buyer is u and neither party will lose from this transaction.

However, if the seller were to send the wrong signals, and of course in this society the buyers have the doubt and are not going to believe them, then there is the probability of loss for both sides. Now let us continue with our model.

The Model

The seller can send two signals (HP and LP), that is high price and low price. For the honest society and with no Gharar transaction, the solution would be (π, u). When the seller signals (HP) since the buyer knows it is the high quality would accept it (A) – the upper left of diagram (1). And when the seller signals (LP), again the buyer accepts it – the lower right of the same diagram.

How about when the seller sends mixed signals, i.e. high and low price for low quality good.

π^s (L, LP, A)= π
π^s (L, HP, A)= π '

In this situation, there is the possibility of loss for at least one side.

The Model

In this model the quality and characteristic of the deal is not known for one side – the buyer– and because of this ignorance, there is the possibility of loss, which can be categorized as a Gharar transaction.

The Model

What would be the consequences of Gharar transaction?

We can show the actions and reactions of the players as follows:

The sellers pursue the honesty strategy (T) and send the correct signals, i.e. high price for high quality good and low price for low quality good. Or they can pursue the opposite strategy (F), i.e. high price for high and low quality goods. On the other hand the buyers can accept (A) any price that the sellers are asking, or reject (R) them and try to acquire extra information. As it was mentioned the buyer always accepts the low price.

The Model

The players' payoffs to the four action pairs are as follows:

(T, A): The sellers get the profit π and the buyers enjoy the utility u.

(F, A): with the probability of (p) the good is the high quality and with the probability of (1-p) it is the low quality good. In this case the expected payoff for the buyers are $pu + (1 - p)u'$ and the expected payoff for the sellers are $p\pi + (1 - p)\pi'$.

The Model

(T,R) : In this case, with the probability of (p) the good has high quality and the buyers have rejected buying it and their utilities are $u'' = u - c$, while with the probability of (1-p) the good has low quality with the utility of u for the buyers. In this case the expected payoff for the buyers are $pu'' + (1 - p)u$. For the sellers, only when the good is low quality they can sell it with the payoff of $(1 - p)\pi$.

The Model

(F,R) : In this situation, the buyers do not accept the sellers' offer, and thus obtain the expected payoff $u'' = u - c$.

The sellers do not get any business, and thus obtain the payoff of 0.

		Buyer	
		$A(q)$	$R(1-q)$
Seller	$T(z)$	π, u	$(1-p)\pi, p(u-c)+(1-p)u$
	$F(1-z)$	$p\pi+(1-p)\pi', pu+(1-p)u'$	$0, u-c$

Nash Equilibrium

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To find the Nash equilibria of this game we can construct the best response functions. The probability of choosing T by the seller is shown by z, and the probability of choosing A by the buyer by q.

Nash Equilibrium

		Buyer $A(q)$	$R(1-q)$
Seller	$T(z)$	π, u	$(1-p)\pi, p(u-c)+(1-p)u$
	$F(1-z)$	$p\pi+(1-p)\pi', pu+(1-p)u'$	$0, u-c$

Seller's best response function: if q=0 (that is the buyer chooses R with the probability of one), then the best response for the seller is z =1, because (1-p)π>0. If q=1 (that is the buyer chooses A with the probability of one), then the best response for the seller is z =0, since π'>π then Pπ+ (1-p)π'>π.

Nash Equilibrium

		Buyer $A(q)$	$R(1-q)$
Seller	$T(z)$	π, u	$(1-p)\pi, p(u-c)+(1-p)u$
	$F(1-z)$	$p\pi+(1-p)\pi', pu+(1-p)u'$	$0, u-c$

Now we want to see for what value of q the seller is indifferent between F and T. For a specific value of q, the expected payoff for the seller if he chooses T, would be , $q\pi+(1-q)(1-p)\pi$, and if he chooses F, would be $q[p\pi+(1-p)\pi']$. Thus, the seller is indifferent between choosing T and F, if:

$$q\pi+(1-q)(1-p)\pi = q[p\pi+(1-p)\pi'] \rightarrow q = \pi/\pi'$$

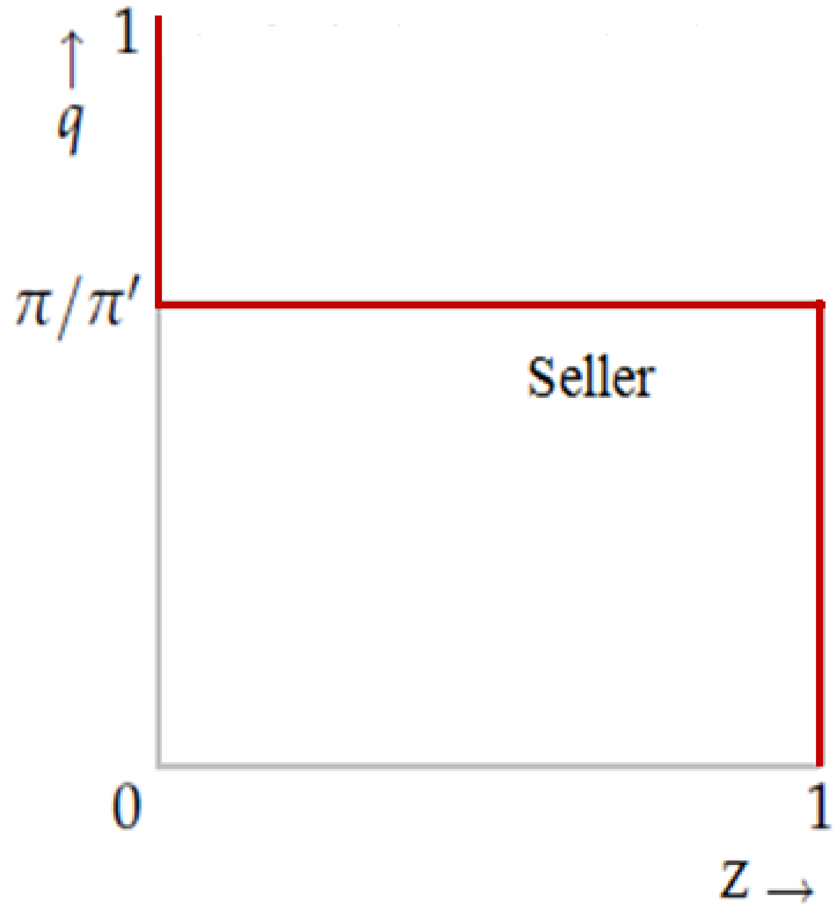


Diagram (2)

Nash Equilibrium

Buyer's best response function: if z=1 (that is the seller chooses T with probability of one), then the buyer's best response would be q=1, since u>u-pc.

if z=0 (that is the seller chooses F with the probability of one), in this case the buyer's best response depends on the values of u-c and $pu+(1-p)u'$.

If $pu+(1-p)u' < u-c$,then the buyer's best response would be q=0.

If $pu+(1-p)u' > u-c$,then the buyer's best response would be q=1.

And if $pu+(1-p)u' = u-c$,then the buyer would be indifferent between choosing A and R.

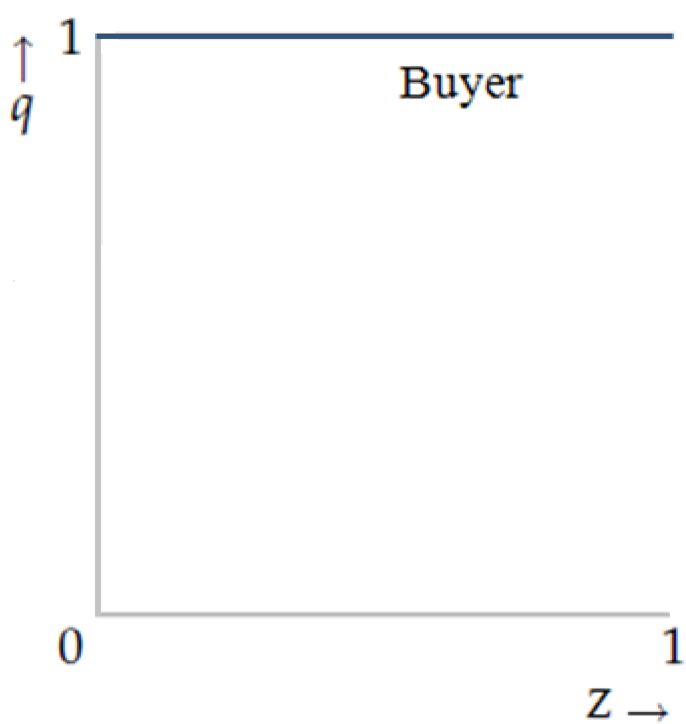
Nash Equilibrium

Thus if $pu + (1-p)u' > u - c$,and z=0 then q=1; and if z=1, then again q=1, which means in this condition the buyer's best response would be q=1 , for all the values of z,; as is shown in the left panel of diagram (3).However, if $pu + (1-p)u' < u - ç$ and z=0 then q=0; and if z=1, then q=1; and the buyer is indifferent between A and R if

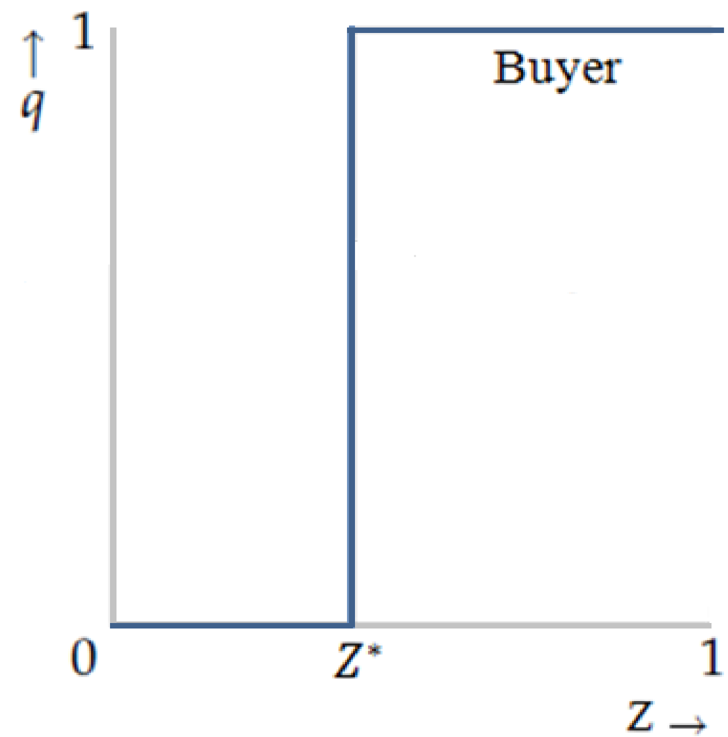
$$zu + (1-z)[pu + (1-p)u'] = z[p(u-c) + (1-p)u] + (1-z)(u-c)$$

$$\rightarrow z = \frac{pu + (1-p)u' - (u-c)}{pu + (1-p)u' - (u-c) - pc} = z^*$$

In this situation, the buyer's best response function takes the form shown in the right panel of diagram (3).



$pu + (1-p)u' > (u-c)$



$pu + (1-p)u' < (u-c)$

Diagram (3)

Equilibrium

Given the best response functions of the players, if $pu + (1-p)u' > u - c$, then the pair of pure strategies (F,A) would be the unique Nash equilibrium of this game.

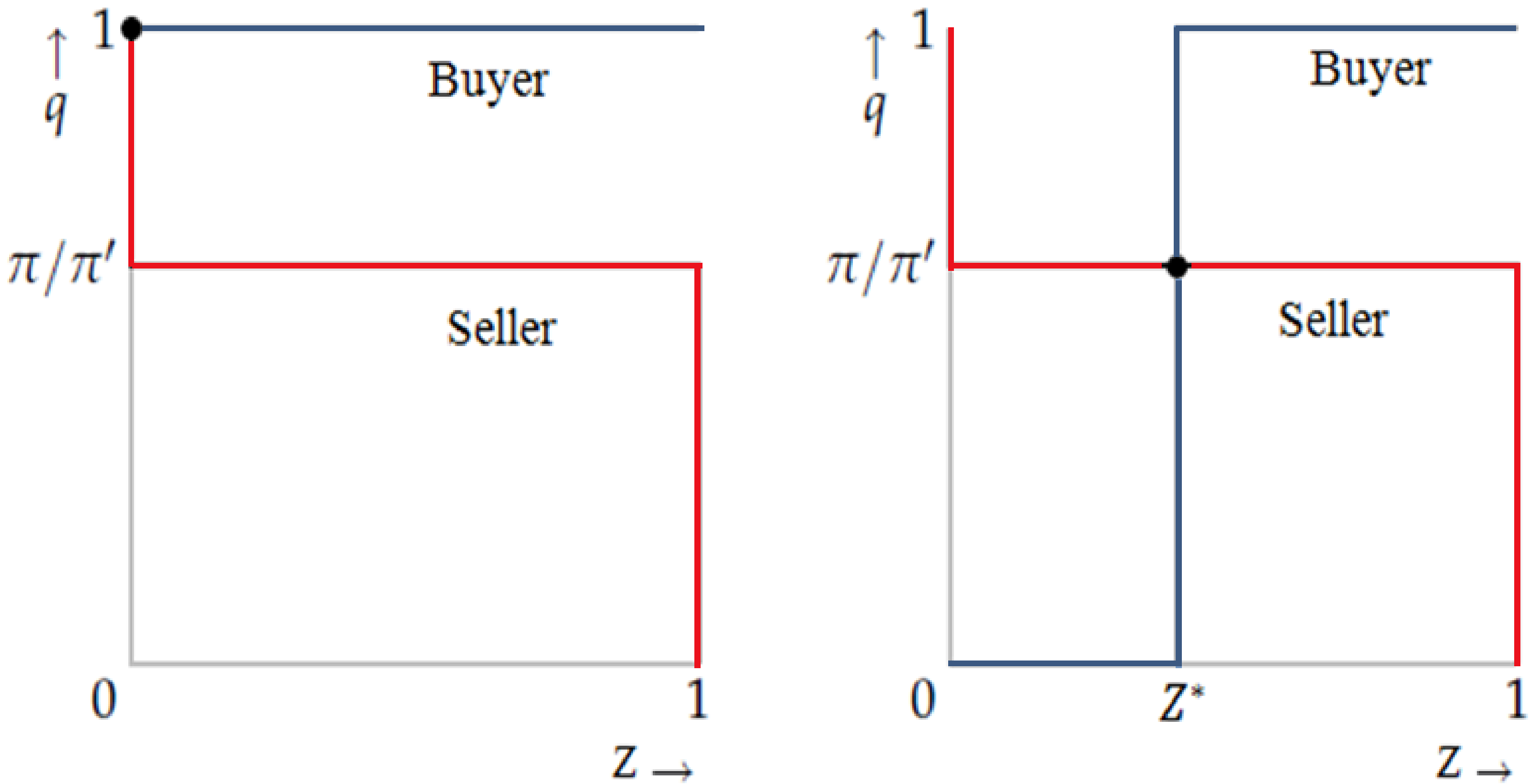
The condition $pu + (1-p)u' > u - c$ means that the expected payoff for the buyer of accepting the seller's price would be more than not accepting it and searching for better prices. This situation could occur when acquiring information would be costly and consequently the disutility of this action is also high. The end result in this situation, as was mentioned, is the pair of pure strategies (F, A); i.e. the seller sends wrong signals and the buyer always accepts them.

Equilibrium

The payoffs for the seller and the buyer are $p\pi + (1-p)\pi'$ and $pu + (1-p)u'$ respectively. Thus, under this condition, the buyer always loses and the seller gets extra profit, however one cannot say anything regarding the total utility of the society (buyers and sellers).

But if $pu + (1-p)u' < u - c$ in this case the unique equilibrium of the game is in mixed strategies, with $(z, q) = (z^*, q^*)$ where

$$z^* = \frac{pu + (1-p)u' - (u-c)}{pu + (1-p)u' - (u-c) - pc}, q^* = \pi/\pi'$$



$pu + (1-p)u' > (u-c)$

$pu + (1-p)u' < (u-c)$

Diagram (4)

Equilibrium

The expected payoffs for the buyer and the seller at the equilibrium are

$$U^B(z^*,q^*) = z^*q^*u + z^*(1-q^*)[p(u-c)+(1-p)u] + (1-z^*)q^*[pu+(1-p)u'] + (1-z^*)(1-q^*)(u-c)$$

$$\rightarrow U^B(z^*,q^*) = u - \frac{cp(u'-u)}{u'-(u-c)}$$

Since $\frac{cp(u'-u)}{u'-(u-c)} > 0$ Then $U^B(z^*,q^*) < u$

		Buyer	
		$A(q)$	$R(1-q)$
Seller	$T(z)$	π, u	$(1-p)\pi, p(u-c)+(1-p)u$
	$F(1-z)$	$p\pi+(1-p)\pi', pu+(1-p)u'$	$0, u-c$

Equilibrium

And for the seller

$$\pi^s(z^*,q^*)=z^*q^*\pi+z^*(1-q^*)(1-p)\pi+(1-z^*)q^*[p\pi+(1-p)\pi']$$

$$\rightarrow \pi^S(z^*,q^*)=\pi-\left[p\pi\left(1-\frac{\pi}{\pi'}\right)\right]$$

Since $\left[p\pi\left(1-\frac{\pi}{\pi'}\right)\right]>0$

Then $\pi^s(z^*,q^*)<\pi$

The (z*,q*) is not a Pareto situation and then not a social optimum.

While, if the quality of the goods are known for both parties – sellers and buyers – which means that the information be transparent and symmetric, or from our point of view there be no Gharar transactions, the payoffs for all would increase and we will have Pareto improvement.

Sensitivity Analysis

If we conduct sensitivity analysis for the equilibrium values:

$$\frac{\partial z^*}{\partial c} = \frac{p(1-p)(u'-u)}{[(1-p)(u'-(u-c))]^2} \to \frac{\partial z^*}{\partial c} < 0$$

$$\frac{\partial q^*}{\partial c} = 0$$

$$\frac{\partial U^B(z^*,q^*)}{\partial c} = -\frac{p(u'-u)^2}{[u'-(u-c)]^2} \to \frac{\partial U^B(z^*,q^*)}{\partial c} < 0$$

$$\frac{\partial \pi^s(z^*,q^*)}{\partial c} = 0$$

Then when c decreases, z^* and $U^B(z^*,q^*)$ increase, in other words when the cost of acquiring information, c, decreases, it is more probable that the sellers are signaling correct information, and consequently the expected payoff for the buyers go up. In this situation the expected payoff of the sellers won’t change and hence the total payoff of the society will increase.

Conclusion

What can be concluded is that by having more transparent society and reducing or removing the problem of asymmetric information which lowers c (information cost), the probability of having Gharar transactions would be reduced and the total utility will increase.

Thank You

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